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## Faculty Working Papers

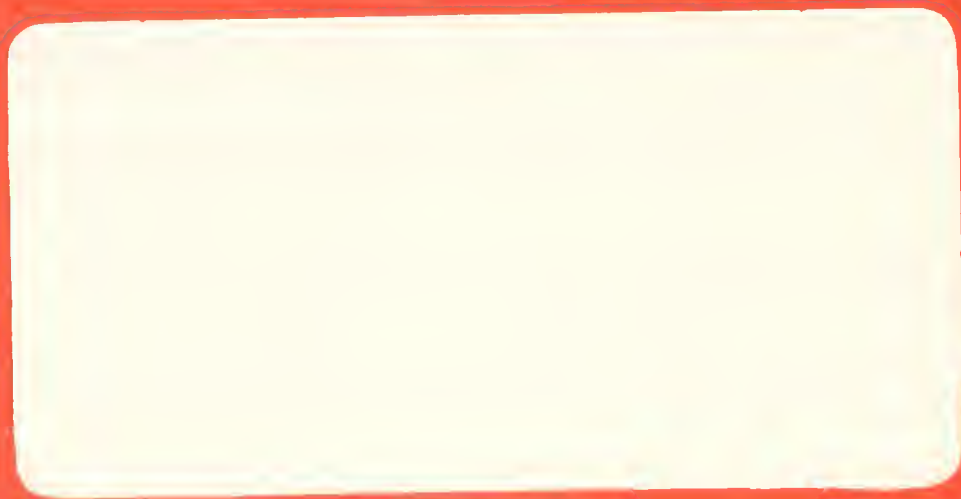
### THE MANY USES OF BOND DURATION

Frank K. Reilly, Professor, Department  
of Finance

Rupinder S. Sidhu, Investment Analyst,  
Prudential Insurance Company

#597

**College of Commerce and Business Administration**  
University of Illinois at Urbana-Champaign



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August 14, 1979

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Summary:

The concept of bond duration was derived in 1938 and "rediscovered" in the early 1970's by several academicians. Since its rediscovery a number of very important uses have been developed. This paper presents the concept and its computation and discusses the several uses in bond analysis, bond portfolio management and common stock analysis.

THE MANY USES OF BOND DURATION\*

Frank K. Reilly

Rupinder S. Sidhu\*\*

INTRODUCTION

During the past five years there has been increasing interest in bond analysis and a rediscovery of a concept in bond analysis originally developed in 1938. Specifically, Professor Frederick Macauley [31] derived a measure of bond term known as duration in 1938 and the concept was generally dormant for almost 30 years until "rediscovered" in the late 1960's. Recently numerous articles have discussed its application to bond analysis and bond portfolio management. The purpose of this paper is to explain the basic concept of duration and discuss in detail how duration is computed including an examination of how duration is affected by maturity, coupon, and market yield. In addition, we consider the main uses of duration in bond analysis (i.e., its relationship to bond price volatility) and bond portfolio management (i.e., how it can be used to "immunize" a bond portfolio). Finally we consider the use of duration in common stock analysis including the problems in its computation and the implications of common stock duration for risk analysis and equity portfolio management.

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\*The authors acknowledge the assistance of Daniel Lehmann and David Wright and comments by Robert Milne and an anonymous referee.

\*\*Professor of Finance, University of Illinois at Urbana-Champaign, and Investment Analyst, Prudential Insurance Company, respectively.



### An Historical Overview

The basic concept of bond duration was derived by Professor Frederick Macaulay in 1938 in a book written for the National Bureau of Economic Research [31]. Notably, the original purpose of duration was as a superior measure of the time pattern of bond flows compared to term to maturity which is the typical measure. Although those familiar with duration generally conceded that it was a superior measure, it was generally ignored for about 30 years. Duration was "rediscovered" in the late 1960s when academicians derived other uses for it. Specifically, Fisher [18] and Hopewell and Kaufman [24] showed that there is a direct relationship between the duration of a bond and its price volatility caused by a change in market interest rates. This direct relationship between duration and bond price volatility has been examined by other authors [5, 9, 11, 12, 21, 22, 33, 34, 41] and shown to be extremely useful to a bond portfolio manager who intends to actively manage his bond portfolio and who attempts to derive superior returns by adjusting his portfolio composition to take advantage of major swings in market interest rates.

Alternatively, assuming a portfolio manager does not want to actively manage his portfolio but is mainly concerned with deriving a specified rate of return that is consistent with the prevailing market returns, Fisher and Weil [17] specified how this can be done by matching the investment horizon of the bond portfolio and the portfolio's duration. This use of duration to "immunize" a bond portfolio has prompted a number of subsequent papers in the last several years [1, 2, 3, 4, 11, 20, 25, 27].

Because of the direct relationship between bond price volatility and duration and the fact that price volatility is considered a measure of risk, some authors have attempted to use duration as a proxy for risk [36]. Given this proxy for risk they have derived capital market lines for bonds relating returns to duration, although there is some question whether duration is an all encompassing measure of risk.

Finally, since duration is basically a measure of the time pattern of returns from an earning asset, there is no reason its use must be limited to bonds. Therefore, the article by Boquist, Racette and Schlarbaum [7] examines the concept applied to bonds and common stock.

Therefore, over time the following uses have been suggested for duration:

1. Superior measure of the time flow of bond returns.
2. An excellent indicator of the expected price volatility for a bond for given changes in market interest rates.
3. A means whereby a bond portfolio can be immunized against changes in market interest rates.
4. Assuming duration is a good proxy for risk, the concept has been used to derive a bond market line and therefore to evaluate bond portfolio performance.
5. As a measure of time flow of returns for common stock.

In the following section these uses are explained and demonstrated.

#### ALTERNATIVE MEASURES OF TIME FLOW OF BOND RETURNS

##### Time Structure of Bond Returns

Although duration is the main subject of this paper, to properly understand the concept, it is useful to place it in perspective with other measures of time structure. Specifically, the whole set of time structure measures are intended to indicate the time flow of returns

from a particular investment instrument--bonds. The measurement of the time flow of returns is important in all investments, but the ability to measure it precisely is typically limited because the analyst is not certain of the timing and size of the flows. Because the cash flow from bonds are specified as to timing and amount, analysts have derived a precise measure of the time flow in contrast to other investments like common stock where the size and timing of the flows are unknown. In this section, each of the principal time flow measures are discussed and demonstrated for two example bonds:

	<u>Bond A</u>	<u>Bond B</u>
Face Value	\$1,000	\$1,000
Maturity	10 years	10 years
Coupon	4%	8%
Sinking Fund	10% a year of face value starting at end of year 5	15% a year of face value starting at end of year 5

#### Term to Maturity

Clearly the most well-known and popular measure of the time flow of returns is term to maturity (TM) which is the number of years prior to the final payment on the bonds. For the two example bonds, the term to maturity is identical--ten years. Term to maturity has the advantage that it is easily identified and easily measured because bonds are always specified in terms of the final maturity date and it is easy to compute the time from the present to that final year. The obvious disadvantage is that this measure ignores the amount and timing of all cash flows except the final payment. For the example bonds it ignores the substantial difference in coupon rates and the difference in the sinking funds.

Weighted Average Term to Maturity

Because the term to maturity ignored all the interim cash flows from a bond, a number of years ago some bond analysts and portfolio managers began computing a time flow measure that considered the interest payments and the final principal payment. Specifically, the weighted average term to maturity (WATM) computes the proportion of each individual payment as a percent of all payments and this proportion becomes the weight for the year (one through ten) the payment is made.<sup>1</sup> It is equal to :

$$WATM = \frac{CF_1(1)}{TCF} + \frac{CF_2(2)}{TCF} + \dots + \frac{CF_n(N)}{TCF}$$

where:  $CF_1$  - cash flow in period 1

(t) - year when cash flow is received

TCF - the total cash flow from the bond.

As an example, the four percent coupon, ten year bond will have total cash flow payments (TCF) of \$1,400 (\$40 a year for ten years plus \$1,000 at maturity). Thus the \$40 payment in year one ( $CF_1$ ) will have a weight of .02857 (\$40/1,400), and each subsequent interest payment will have the same weight. The principal payment in year ten has a weight of .74286 (\$1,000/1,400). The specific computation of the weighted average term to maturity for the two bonds is demonstrated in Table 1. Two points are notable. First, the WATM is definitely less than the term to maturity because it takes account of all the interim flows in addition to the final principle payment. Second, the bond with the larger coupon has a shorter WATM because a larger proportion of its total cash flows are

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<sup>1</sup> Although it is recognized that interest payments are typically made at six month intervals, we assume annual payments at the year end to simplify the computations.

TABLE 1

COMPUTATION OF WEIGHTED AVERAGE TERM TO MATURITY  
(Assumes annual interest payments)

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(1) Year	(2) Cash Flow	Bond A	
		(3) Cash Flow/T.C.F	(4) (1) x (3)
1	\$ 40	.02857	.02857
2	40	.02857	.05714
3	40	.02857	.08571
4	40	.02857	.11428
5	40	.02857	.14285
6	40	.02857	.17142
7	40	.02857	.19999
8	40	.02857	.22856
9	40	.02857	.25713
10	1040	.74286	7.42860
Sum	<u>\$1400</u>	<u>1.00000</u>	<u>8.71425</u>

Weighted Average Term  
to Maturity = 8.71 Years

Bond B			
(1)	(2)	(3)	(4)
1	\$ 80	.04444	.04444
2	80	.04444	.08888
3	80	.04444	.13332
4	80	.04444	.17776
5	80	.04444	.22220
6	80	.04444	.26664
7	80	.04444	.31108
8	80	.04444	.35552
9	80	.04444	.39996
10	1080	.60000	6.00000
Sum	<u>\$1800</u>	<u>1.00000</u>	<u>7.99980</u>

Weighted Average Term  
To Maturity = 8.00 Years

derived from the coupon payments that come prior to the final principle payment at maturity. Specifically, for the 4 percent bond, the interest payments constitute 28.6 percent (400/1,400) of the total returns, while for the 8 percent bond the interest payments make up 44.4 percent (800/1,800) of the total flow. Obviously it is also possible to compute a measure of the time flow of returns including the sinking fund payments and the WATM would be even lower. This computation is discussed in a subsequent section.

A major advantage of the WATM measure is that it considers the timing of all the flows from the bond rather than only the final payment. A drawback of this time flow measure is that it does not consider the time value of the flows. Note that the interest payment in the first year has the same weight as the interest payment in the tenth year, although the present value of the payment in year ten is substantially less. Also, the \$1,000 principle would have the same weight whether it was made in year ten or year twenty.

### Duration

The duration measure is similar to the WATM with the one exception that all flows are in terms of present value. Specifically, duration is equal to:

$$D = \frac{\sum_{t=1}^n \frac{C_t(t)}{(1+r)^t}}{\sum_{t=1}^n \frac{C_t}{(1+r)^t}}$$

where:  $C_t$  = interest and/or principal payment at time  $t$   
 $(t)$  = length of time to the interest and/or principal payment

n = length of time to final maturity

r = yield to maturity

Similar to the WATM, this could be set forth as

$$D = \frac{PVCF_1(1)}{PVTCF} + \frac{PVCF_2(2)}{PVTCF} + \dots + \frac{PVCF_n(n)}{PVTCF}$$

where:  $PVCF_i$  = present value of the cash flow in period i discounted at current yield to maturity.

(t) = period when cash flow is received

PVTCF = present value of total cash flow from the bond discounted at current yield to maturity. Obviously this is the prevailing market price for the bond.

The computation of the duration for the two example bonds is contained in Table 2. As noted, this measure is very similar to the WATM except that all flows are in terms of present value. Therefore, duration is simply a weighted average maturity stated in present value terms. Specifically, the time in the future a cash flow is received is weighted by the proportion that the present value of that cash flow contributes to the total present value or price of the bond. Again it is assumed that interest payments are made annually. The use of the more realistic semi-annual payments would result in a shorter duration (7.99 years versus 8.12 years and 7.07 years compared to 7.25 years).

Similar to WATM, the duration of the bond is shorter than the term to maturity because of the interim interest payments. Obviously if there were no interim payments (zero coupon), the duration, the WATM and the term to maturity would be the same because there would only be a single payment at maturity so that 100 percent of the total cash flow or present value of cash flow would come at maturity. Also similar to the WATM, the duration is inversely related to the coupon for the bond--i.e.,

TABLE 2

COMPUTATION OF DURATION FOR EXAMPLE BONDS ASSUMING 8 PERCENT MARKET YIELD

(1) Year	(2) Cash Flow	(3) P.V. @ 8%	<u>Bond A</u> (4) P.V. of Flow	(5) P.V. % of Price	(6) (1) x (5)	
1	\$ 40	.9259	\$ 37.04	.0506	.0506	
2	40	.8573	34.29	.0469	.0938	
3	40	.7938	31.75	.0434	.1302	
4	40	.7350	29.40	.0402	.1608	
5	40	.6806	27.22	.0372	.1860	
6	40	.6302	25.21	.0345	.2070	
7	40	.5835	23.34	.0319	.2233	
8	40	.5403	21.61	.0295	.2360	
9	40	.5002	20.01	.0274	.2466	
10	1040	.4632	481.73	.6585	6.5850	Duration =
Sum			<u>\$731.58</u>	<u>1.0000</u>	<u>8.1193</u>	8.12 Years
<u>Bond B</u>						
1	\$ 80	.9259	\$ 74.07	.0741	.0741	
2	80	.8573	68.59	.0686	.1372	
3	80	.7938	63.50	.0635	.1906	
4	80	.7350	58.80	.0588	.1906	
5	80	.6806	54.44	.0544	.2720	
6	80	.6302	50.42	.0504	.3024	
7	80	.5835	46.68	.0467	.3269	
8	80	.5403	43.22	.0432	.3456	
9	80	.5002	40.02	.0400	.3600	
10	1080	.4632	500.26	.5003	5.0030	Duration =
Sum			<u>\$1000.00</u>	<u>1.0000</u>	<u>7.2470</u>	7.25 Years



the larger the coupon, the greater the proportion of total returns received in the interim, and the shorter the duration. Figure 1 contains a graph of the relationship between duration and maturity for alternative coupons.

A final variable that can affect the duration of a bond that does not influence the WATM, is the prevailing market yield (r). The market yield does not influence WATM because WATM does not consider the present value of flows. Obviously, the market yield affects both the numerator and the denominator of the duration computation, but it affects the numerator more. As a result, there is an inverse relationship between a change in the market yield and a bond's duration--i.e., an increase in the market yield will cause a decline in duration, all else the same. The effect of such a change can be seen for the two example bonds when different market yields are considered and semi-annual payments are assumed.

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TABLE 3

DURATION FOR EXAMPLE BONDS ASSUMING ALTERNATIVE  
MARKET YIELDS AND SEMI-ANNUAL INTEREST PAYMENTS

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	<u>Market Yields</u>			
	<u>0%</u>	<u>4%</u>	<u>8%</u>	<u>12%</u>
Bond A	8.60*	8.34	7.99*	7.59
Bond B	7.87*	7.50	7.07*	6.61

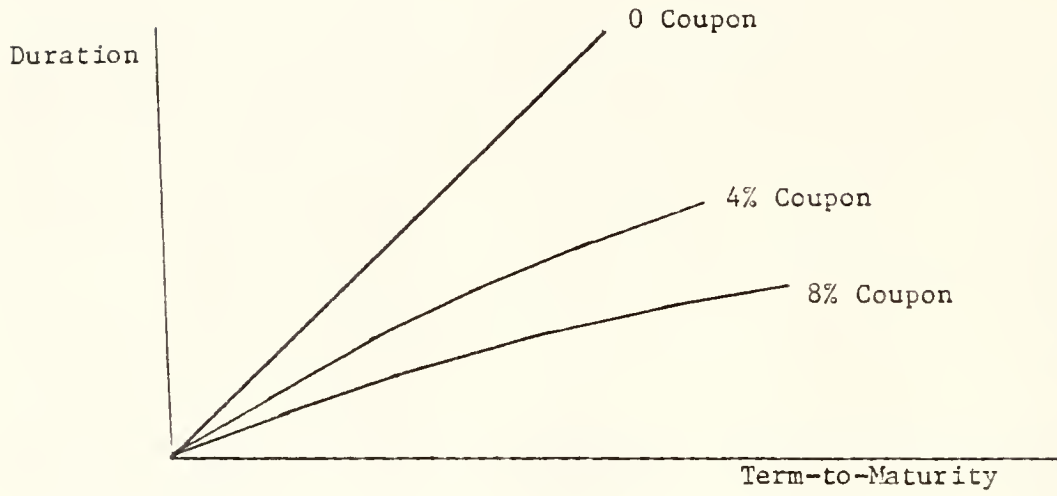
\*These duration figures differ from Table 1 and Table 2 due to the use of semi-annual interest payments.

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These results indicate that there clearly is an impact from different market yields although the effect is not overpowering. In addition,

FIGURE 1

RELATIONSHIP BETWEEN DURATION AND  
TERM TO MATURITY FOR ALTERNATIVE COUPONS



the inclusion of the zero market yield indicates the relationship between duration and the WATM--at a zero market yield duration is the same as WATM because there is no discounting.

#### Effect of Sinking Funds

The discussion thusfar has considered the interest and principal payments for the bond but has ignored the effects of sinking funds which could be important because a large proportion of current bond issues have sinking funds that definitely have an effect on a bond's duration. The computation of the duration for the bonds with the sinking funds is contained in Table 4. As shown, the consideration of the sinking fund caused the computed duration to decline by approximately one year in both cases (i.e., from 8.12 to 7.10 for Bond A, and from 7.25 to 6.21 for Bond B).

Notably, the effect of the sinking fund on the time structure of cash flows for the bond is certain to the issuer of the bond since the firm must make these payments. Hence this legal cash flow requirement definitely affects the firm's cash flow requirements. In contrast, the sinking fund may not affect the investor because the money put into the sinking fund may not necessarily be used to retire outstanding bonds, or even if it is, it is not certain that a given investor's bonds will be called for retirement.

#### Effect of Call on Duration

In contrast to the sinking fund that may only affect a few investors and only reduces the duration by one year, the effect of a bond being called will affect all bondholders and the impact on duration can be

TABLE 4

COMPUTATION OF DURATION FOR EXAMPLE BONDS ASSUMING 8 PERCENT  
MARKET YIELD AND CONSIDERING SINKING FUND

(1) Year	(2) Cash Flow	(3) P.V. @ 8%	Bond A		(6) (1) x (5)
			(4) P.V. of Flow	(5) P.V./Total C.F.	
1	\$ 40	.9259	\$ 37.04	.04668	.04668
2	40	.8573	34.29	.04321	.08642
3	40	.7938	31.75	.04001	.12003
4	40	.7350	29.40	.03705	.14820
5	140	.6806	95.28	.12010	.60050
6	140	.6302	88.23	.11119	.66714
7	140	.5835	81.69	.10295	.72065
8	140	.5403	75.64	.09533	.76264
9	140	.5002	70.03	.08826	.79434
10	540	.4632	250.13	.31523	3.15230
Sum			<u>\$793.48</u>	<u>1.00000</u>	<u>7.09890</u> Duration 7.10 Years

Bond B					
(1) Year	(2) Cash Flow	(3) P.V. @ 8%	(4) P.V. of Flow	(5) P.V./Total C.F.	(6) (1) x (5)
1	\$ 80	.9259	\$ 74.07	.06778	.06778
2	80	.8573	68.59	.06276	.12552
3	80	.7938	63.50	.05811	.17433
4	80	.7350	58.80	.05380	.21520
5	230	.6806	156.54	.14324	.71620
6	230	.6302	144.95	.13264	.79584
7	230	.5835	134.21	.12281	.85967
8	230	.5403	124.27	.11371	.90968
9	230	.5002	115.05	.10528	.94752
10	330	.4632	152.86	.13987	1.39870
Sum			<u>\$1092.84</u>	<u>1.00000</u>	<u>6.21044</u> Duration 6.21 Year

substantial. To show the impact, consider the following example: 30 year bond, 8 percent coupon, selling at par, callable after 10 years at 108.

First, it is necessary to compute a cross-over yield.<sup>2</sup> At yields above the cross-over yield the yield to maturity is the minimum yield. When the price of the bond rises to some value above the call price and the market yield declines to a value below the cross-over yield, the investor should use the yield to call for the minimum yield. Put another way, at this price and yield there is a high probability the firm will exercise the call option when it is available. As shown by Homer and Leibowitz [23], it is possible to calculate the cross-over yield by deriving the yield to maturity for a bond selling at the call price for the original maturity minus the years of call protection--i.e., in the current example this would involve deriving the YTM for an 8 percent coupon bond selling at 1080 maturing in 20 years (the implied cross-over yield is 7.24 percent).

One Year Later: Maturity = 29 years; Years to call = 9 years. Let us assume that market rates decline to the point where the YTM for the example bond is 7 percent which is below the cross-over yield of 7.24 percent. At this price (\$1123.43), the YTC is 6.2 percent. If a bond portfolio manager ignored the call option and computed the duration of this bond to maturity (29 years) assuming a market yield of 7 percent, the duration would be 12.49 years. In contrast, if one recognized the call option and computed the duration for a bond to be called in 9 years at

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<sup>2</sup>This discussion of cross-over yield and its computation is drawn from Homer and Leibowitz [23, pp. 58-63].

a price of \$1080 and used the yield to call of 6.2 percent, the duration would be 6.83 years. (This example is summarized in Table 5.)

The point is, the existence of a call option, which is almost universal on corporate bonds, can have a dramatic impact on the computed duration for the bond. In the example we assumed a deferred call of 10 years which is currently the maximum period compared to the more typical 5 year deferrment.

#### Duration of GNMA Bonds

During the past several years there has been a substantial increase in investor interest in GNMA pass-through bonds because of the inherent safety of the bonds and the higher yields compared to other government securities. Without detracting from the safety and yield characteristics of these securities, a portfolio manager should recognize the extreme difference between the initial promised term-to-maturity, the empirical maturity, and the probable duration taking into account the form of cash flow and the empirical maturity. It is well recognized that an investor in a GNMA pass through is basically purchasing a share of a pool of mortgages. As a result, each month the investor receives a payment from the mortgages that includes not only interest, but also partial repayment of the principal. In addition, if a homeowner subsequently decides to acquire another home because he is moving or for other reasons, he will naturally sell his current home and pay off his mortgage. This results in numerous prepayments on mortgages. As a result, mortgage contracts are like bonds with sinking funds because they pay interest and principal over time and they are also like bonds that are freely callable (the prepayment penalty is generally waived if

TABLE 5

EXAMPLE SHOWING IMPACT OF CALL OPTIONS ON COMPUTED DURATION

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Original Bond: 8 percent coupon bond sold at par with 30 years to maturity. Callable in 10 years at 108 of par.  
(Computed cross-over yield is 7.24 percent.)

One Year Later: Market yields on bond decline from 8 percent to 7 percent.

Current market price:	\$1123.43
Yield to maturity (29 years):	7%
Yield to call (9 years):	6.2%
Call price:	108

Duration: At 7% yield and 29 years maturity -- 12.49 years

At 6.2% yield, 9 years to call at 108 -- 6.83 years

you sell the house to buy another house) if they can be paid off when the house is sold. Taking account of both of these characteristics means that the empirical duration of a GNMA pass through is substantially less than the stated maturity.

As an example, the stated maturity of most home mortgages is 25 years. Given the nature of the payment stream which includes principle and interest, the duration of a mortgage without prepayment is substantially less than the stated maturity. As examples, assuming a 10 per cent market rate and annual payments at the end of the year, a 30 year mortgage has a duration of 9.18 years; a 25 year mortgage has a duration of 8.46 years and a 20 year mortgage has a duration of 7.51 years (the consideration of realistic monthly payments would reduce these durations further). In addition, because of the numerous prepayments, it is acknowledged that the empirical maturity of most mortgage pools is actually only about 12 years rather than the stated 25 years. Therefore, if one assumes the principle and interest payments for 12 years and a prepayment at the end of 12 years (with no call premium), the computed durations would decline further (e.g., under these assumptions the mortgages have the following durations: 30 years--7.22 years; 25 years--7.04 years; 20 years--6.71 years). As stated, bond portfolio managers should recognize that they are acquiring relatively short duration bonds when they invest in these securities.

#### Summary of Duration Properties

The prior discussion indicated the usefulness of duration as a measure of the time structure of flows for bonds. The major properties of duration are:

- duration is positively related to the maturity of the bond except for very long maturity bonds selling at a discount (for a discussion of this point see Van Horne [36, p. 120]).



- duration is inversely related to the coupon on a bond.
- duration is inversely related to the market yield for the bond.
- a bond's duration is reduced by a sinking fund provision.
- a bond's duration can be substantially reduced by a call provision.

#### Relationship Between Time Flow Measures

As noted previously, the WATM and duration for a bond will be equal to its term to maturity in cases where the coupon rate is zero--i.e., there are no interim cash flows prior to maturity. Also, these are the maximum limits for both these measures--i.e., the WATM and duration for a bond will never exceed its term to maturity. In fact, Fisher and Weil [17] suggest that the way for insurance companies to get long duration portfolios that will match their long-term liabilities is to encourage some issuers (including the government) to sell long term zero coupon discount bonds that would have maturities and duration of 30 or 40 years. With coupons of almost any size it is nearly impossible to find bonds that have durations in excess of 20 years and most bonds have a limit of about 15 years.

As shown in the examples in Table 1 and 2, the WATM is always longer than the duration of a bond and the difference increases with the market rate used in the duration formula. This is consistent with the observation that there is an inverse relationship between duration and the market rate. Further, this relationship leads to the observation that the WATM and duration for a bond are equal when the market rate is zero.

#### DURATION AND BOND PRICE VOLATILITY

##### Form of Relationship

As noted in the introduction, one of the reasons for an increased interest in the concept of duration is that it takes account of the effect

of all payments when specifying the time structure of returns. Another characteristic of duration that caused a renewed interest in the concept of duration was the recognition that there is a direct relationship between the duration of a bond and the price volatility for the bond assuming a given change in market rates of interest. This property was recognized by Macauley [31] and Fisher [18] and the specific form of the relationship was set forth in a paper by Hopewell and Kaufman [24]. The specific relationship is:

$$\% \Delta \text{Bond Price} = -D^*(\Delta r)$$

where:  $\% \Delta \text{Price}$  = the percent change in price for the bond

$D^*$  = the adjusted duration of the bond in years which is equal to  $D/(1 + r)$ .

$\Delta r$  = the change in the market yield in basis points divided by 100 (e.g., a 50 basis point decline would be  $-.5$ )

As an example, assume a bond has a duration of 10 years, an adjusted duration of 9.259 years ( $10/1.08$ ) and interest rates go from 8 percent to 9 percent. Then:

$$\begin{aligned} \% \Delta \text{Bond Price} &= -9.259(100/100) \\ &= -9.259(1) \\ &= -9.259\% \end{aligned}$$

In this example, the price of the bond should decline by about 9.3 percent for every one percent (100 basis point) increase in market rates. For most practical cases investors tend to use the unadjusted duration figure when computing the impact. At high duration figures and "reasonable" market rates, the difference is relatively minor. The important point is, the longer the duration of a bond (or a portfolio of bonds) the greater the price volatility of the bond (or bond portfolio) for a

change in interest rates--i.e., there is a very direct relationship between duration and interest rate risk. Notably, the duration of a portfolio is simply the weighted average of the duration of the individual securities in the portfolio where the weights are relative market values.

#### Implications for Portfolio Management

This direct relationship between duration and interest rate sensitivity is important to an active bond portfolio manager who attempts to derive superior returns by adjusting the composition of his portfolio to benefit from swings in market rates of interest. Assuming this portfolio philosophy, the idea is to construct a bond portfolio with maximum interest rate sensitivity prior to a period when the portfolio manager expects a decline in interest rates and vice versa during a period of rising interest rates. The point is, assuming the portfolio manager expects a decline in interest rates, the portfolio should be constructed with the maximum duration rather than considering only term to maturity because duration is a superior indicator of the interest sensitivity of the portfolio. The point is, when the forecast is that rates are declining and it is decided to increase the average duration of your portfolio to derive the maximum price changes from the interest rate change, an awareness of duration and the factors that influence it, would mean you would be conscious of coupon, call features, and sinking funds in addition to maturity in determining shifts in the portfolio composition. Therefore, this property of duration means that it is a useful concept for the active bond portfolio manager. For a discussion of some of the practical aspects of implementating this use of duration see the series of articles by Diller [11].

## DURATION AND IMMUNIZATION

### Components of Interest Rate Risk

A major problem encountered in bond portfolio management is deriving a given rate of return to satisfy an ending wealth requirement at a future specific date—i.e., the investment horizon. If the term structure of interest rates was flat and the level of market rates never changed between the time of purchase and the future specific date when the funds were required, it would be possible to acquire a bond with a term to maturity equal to the desired investment horizon and the ending wealth from the bond purchase would equal the promised wealth position implied by the promised yield to maturity. Specifically, the ending wealth position would be the beginning wealth times the compound value of a dollar at the promised yield to maturity. Unfortunately, in the real world the term structure of interest rates is not typically flat and the level of interest rates is constantly changing. Because of changes in the shape of the term structure and changes in the level of interest rates, the bond portfolio manager faces what is referred to as "interest rate risk" between the time of investment and the future target date. Specifically, interest rate risk can be defined as the uncertainty regarding your ending wealth position due to changes in market interest rates between the time of purchase and the target date. In turn, interest rate risk is composed of two risks which are a price risk and a coupon reinvestment risk. The price risk occurs because if interest rates change prior to the target date and the bond is sold prior to maturity, the market price for the bond (i.e., the "realized" price) will differ from the expected price assuming there had been no change in rates.

Obviously if rates increased since the time of purchase, the realized price for the bond in the secondary market would be below expectations, while if interest rates declined the realized price would be above expectations.

The coupon reinvestment risk arises because the yield to maturity computation implicitly assumes that all coupon flows will be reinvested to yield the promised yield to maturity (for a detailed elaboration of this point, see Homer and Leibowitz [23]). Obviously if subsequent to the purchase of the bond, interest rates decline, it will not be possible to reinvest the coupon cash flows at the promised yield to maturity, but they will be reinvested at lower rates and the ending wealth would be below expectations. In contrast, if interest rates increase, the interim cash flows will be reinvested at rates above expectations and the ending wealth would be above expectations.

#### Immunitization and Interest Rate Risk

Note that the price risk and the reinvestment risk derived from a change in interest rates have an opposite effect on the investor's ending wealth position. Specifically, an increase in the level of market interest rates will cause an ending price that is below expectations, but the reinvestment of interim cash flows will be at a rate above expectations so this reinvestment income will be above expectations. In contrast, a decline in market interest rates will provide a higher than expected ending price, but lower than expected ending wealth from the reinvestment of interim cash flows. It is clearly important to a bond portfolio manager with a specific target date (i.e., known holding period) to attempt to eliminate these two risks derived from changing interest

rates. The elimination of these risks from a bond portfolio is referred to as immunization. This concept is discussed in Redington [35] and is defined by Fisher and Weil [17, p. 415] as follows:

A portfolio of investments in bonds is immunized for a holding period if its value at the end of the holding period, regardless of the course of interest rates during the holding period, must be at least as large as it would have been had the interest-rate function been constant throughout the holding period.

If the realized return on an investment in bonds is sure to be at least as large as the appropriately computed yield to the horizon, then that investment is immunized.

Previously in the Fisher and Weil paper there is an analysis of the promised yields on bonds for the period 1925-1968 compared to the realized returns on bonds. This presentation demonstrates the difference between the promised yield and the realized yield and indicates the importance of being able to immunize a bond portfolio. It is shown that it is possible to immunize a bond portfolio if you can make one assumption. The required assumption is that if the interest rate function shifts, that the change in interest rates is the same for all future rates. Somewhat more technically, the assumption says that if forward interest rates change, all rates change by the same amount. Given this assumption it is proven by Fisher and Weil that a portfolio of bonds is immunized from the interest rate risk if the duration of the portfolio is equal to the desired investment horizon. As an example, if the desired holding period of a bond portfolio is eight years, in order to immunize the portfolio, the duration of the bond portfolio should be set equal to eight years. In order to have a portfolio with a given duration, the weighted average duration (with weights equal to

the proportion of value) is set at the desired length following an interest payment and then all subsequent cash flows are invested in securities with a duration equal to the remaining horizon value.

The whole point of the proof of the immunization theorem by Fisher and Weil is that the two risks discussed (price risk and reinvestment rate risk) are affected differently by a change in market rates--i.e., when the price change is positive the reinvestment change will be negative and vice versa. The crucial question as regards immunization is, when will these two components of interest rate risk be equal so that they offset each other? Fisher and Weil proved that duration was the time period at which the price risk and the coupon reinvestment risk of a bond portfolio are of equal magnitude but opposite in direction. This is also noted and discussed in [1, 2, 3, 4, 11, 27, 28, 34, 38].

#### Application of the Immunization Principle

Following a statement and discussion of the theorem regarding immunization and duration, Fisher and Weil carried out a simulation to show the effects of attempting to apply the immunization concept in the real world, compared to a naive portfolio strategy where the portfolio's maturity was set equal to the investment horizon--i.e., if the investment horizon was eight years, the average term-to-maturity of the portfolio would be set at eight years rather than the duration set at eight years (obviously, assuming coupon bonds the duration of the portfolio with an average maturity of eight years would be shorter than eight years). The simulation computed the ending wealth ratios for alternative investment horizons (5, 10, and 20 years) assuming: (1) the expected yield was realized (the yield curve never shifted), (2)

the portfolio was constructed so that the duration was equal to the investment horizon (i.e., the duration strategy), and (3) the portfolio's maturity was equal to the investment horizon (i.e., the naive maturity strategy). The analysis involved a comparison of the ending wealth ratio for the duration strategy portfolio and the naive maturity strategy portfolio to the wealth ratio assuming no change in the interest rate structure. The point is, if a portfolio was perfectly immunized, the actual ending wealth should be equal to the expected ending wealth implied by the promised yield. Therefore, these comparisons should indicate which portfolio strategy does a superior job of immunization. It was shown that the duration strategy results were consistently closer to the expected promised yield results, although the results were not perfect (i.e., the duration portfolio was not perfectly immunized). The difference was because the basic assumption was not always true--when interest rates change, all interest rates did not change by the same amount. The authors concluded that the naive maturity strategy removes the majority of the uncertainty of the expected wealth ratio from a long-term bond portfolio, and most of the remaining uncertainty is removed when the duration strategy is employed. The authors contend that the reduction in the standard deviation of the duration strategy portfolio was so dramatic that one is led to conclude that a properly chosen portfolio of long-term bonds (based upon matching the investment horizon with duration) is essentially riskless.

A subsequent note by Bierwag and Kaufman [1] points out that there are several specifications of the duration measure. The measure derived by Macauley [31], that is used throughout this paper, discounts all flows



by the prevailing average yield to maturity on the bond being measured. Alternatively, Fisher and Weil [17] define duration using future one period discount rates (forward rates) to discount the future flows. Depending upon the shape of the yield curve the two definitions could give different answers. If all forward rates are equal so that the yield curve is flat, the two definitions will compute equal durations. After likewise demonstrating that the way to immunize a portfolio is to match duration and the investment horizon, Bierwag and Kaufman noted that the definition of duration used should be a function of the nature of the shock to the interest rate structure. Specifically, it is possible to conceive of an additive shock to interest rates where all interest rates are changed by the same nominal amount (e.g., 50 basis points). Alternatively, the interest rate shock could be multiplicative, where all interest rates change by the same percent (e.g., all rates decline by 10 percent). It is then contended (and proven in Bierwag [2]) that the optimal definition of duration used to perfectly immunize a portfolio will depend upon the nature of the shock to the interest rate structure. In the case of an additive shock the Fisher-Weil definition is best, while a third definition of duration is best if the shock is multiplicative. The authors compute the duration for a set of bonds using the three definitions of duration ( $D_1$  - Macauley;  $D_2$  - Fisher-Weil;  $D_3$  - Bierwag-Kaufman) and conclude [1, p. 367]:

Except at high coupons and long maturities, the values of the three definitions do not vary greatly. Thus,  $D_1$  may be used as a first approximation for  $D_2$  and  $D_3$ . The expression for  $D_1$  has the additional advantage of being a function of the yield to maturity of the bond. As a result, neither a forecast of the stream of one-period forward rates over the maturity of the bond nor a specific assumption about the nature of the random shocks is required.

Example of Immunization

An example of the effect of attempting to immunize a portfolio by matching the investment horizon and the duration of a bond portfolio is contained in Table 6 using a single bond. It is assumed that the portfolio manager's investment horizon is eight years and the current yield to maturity for eight year bonds is 8 percent. Therefore, the ending wealth ratio for an investor should be  $1.8509[(1.08)^8]$  which should be the ending wealth ratio for a completely immunized portfolio. The example considers two portfolio strategies--the maturity strategy where the term to maturity is set at eight years, and the duration strategy where the duration is set at eight years. For the maturity strategy it is assumed that the portfolio manager acquires an eight year 8 percent bond. In contrast, for the duration strategy it is assumed the portfolio manager acquires a ten year, 8 percent bond which has approximately an eight year duration (8.12 years) assuming an 8 percent yield to maturity (see Table 2). It is further assumed that there is a single shock to the interest rate structure at the end of year four and the market yield goes from 8 percent to 6 percent and remains at 6 percent through year eight.

As shown, due to the interest rate change the wealth ratio for the maturity strategy bond is below the desired wealth ratio because of the shortfall in the reinvestment cash flow after year four (i.e., the interim coupon cash flow is reinvested at 6 percent rather than 8 percent). Note that the maturity strategy eliminated the price risk because the bond matured at the end of year eight. Alternatively, the duration strategy portfolio likewise suffered a shortfall in reinvest-

TABLE 6

AN EXAMPLE OF THE EFFECT OF A CHANGE IN  
MARKET RATES ON A BOND (PORTFOLIO) THAT USES THE  
MATURITY STRATEGY VERSUS THE HORIZON STRATEGY

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Year	<u>Results with Maturity Strategy</u>			<u>Results with Horizon Strategy</u>		
	Cash Flow	Reinv. Rate	End Value	Cash Flow	Reinv. Rate	End Value
1	\$ 80	.08	\$ 80.00	\$ 80	.08	\$ 80.00
2	80	.08	166.40	80	.08	166.40
3	80	.08	259.71	80	.08	259.71
4	80	.08	360.49	80	.08	360.49
5	80	.06	462.12	80	.06	462.12
6	80	.06	596.85	80	.06	596.85
7	80	.06	684.04	80	.06	684.04
8	1080	.06	1805.08	1120.684*	.06	1845.72

Expected Wealth Ratio - 1.8509

\*The bond could be sold at its market value of \$1,040.64 which is the value for an 8 percent bond with two years to maturity priced to yield 6 percent.

ment cash flow because of the change in market rates. Notably, this shortfall due to the reinvestment risk is offset by an increase in the ending value for the bond due to the decline in market rates (i.e., the bond is sold at the end of year eight at 104.06 because it is an 8 percent coupon bond with two years to maturity selling to yield 6 percent).

Note that if market interest rates increased during this period that the maturity strategy portfolio would have experienced an excess of reinvestment income compared to the expected cash flow, and the wealth ratio for this strategy would have been above expectations. In contrast, in the duration portfolio the excess cash flow from reinvestment under this assumption would have been offset by a decline in the ending price for the bond. While under these latter assumptions the maturity strategy would have provided a higher than expected ending value, the whole purpose of immunization was to eliminate uncertainty (i.e., have the realized wealth position equal the expected wealth position) which is what is accomplished with the duration strategy.

In summary, it has been shown that the concept of duration is important to the bond portfolio manager with a specified investment horizon attempting to reduce the interest rate risk from his long-term bond portfolio--i.e., the portfolio manager does not want to attempt to predict future market rates, but simply wants to derive a specified return irrespective of future rates. It is shown that the two components of interest rate risk (price risk and reinvestment rate risk) are opposite in sign and will exactly offset each other if the portfolio's duration is set equal to the investment horizon. Although there are some limiting assumptions regarding the nature of the change in the interest

rate structure, Fisher and Weil showed that a real world simulation of the technique derives results that have very small deviations from what expectations would be with complete immunization. It is demonstrated that a substantial portion of the interest rate risk is eliminated with the maturity strategy (because, by definition, the price risk is eliminated) and even more risk is eliminated with the duration strategy because the price risk is allowed to offset the reinvestment rate risk.

## YIELD CURVES AND BOND MARKET LINES

### Derivation of Yield Curves

The typical yield curve is derived by plotting the yield to maturity (on the vertical axis) against the term to maturity (on the horizontal axis) for bonds of equal risk. Hopewell and Kaufman [24] contend that this practice can result in abnormal curves if the bonds used have significantly different coupons. The point is, it is entirely possible to conceive of two bonds with different terms to maturity but the longer maturity bond will have the shorter duration if the coupons are different--e.g., a 20 year maturity bond with a large coupon could have a shorter duration than an 18 year bond with a small coupon. Therefore, it is suggested by Carr, Halpern and McCallum [10] that yield curves should be constructed with yield to maturity on the vertical axis and duration on the horizontal axis. Further, they contend that forward rates (future implied short-term rates) should be computed on the basis of the duration yield curve. Note that, it is still necessary that the yield curves be derived using bonds of equal risk--e.g., all government bonds or all AAA rated bonds.

An example of a yield curve for a sample of Government bonds using term to maturity and duration is contained in Table 7 and plots of the two yield curves are contained in Figure 2 and 3. This example indicates the difference in the two curves. Clearly the duration-yield curve is much shorter than the maturity-yield curve and any slope (up or down) would be much sharper.

#### Duration and a Bond Market Line

Because bond duration is an indicator of bond price volatility one can conceive of duration as a useful risk proxy for bonds (the shortcoming of average term-to-maturity in this regard is discussed in [43, 44]). Specifically, with an increase in duration a bond is more volatile for a given change in market interest rates, all else the same. Therefore, if one were to consider the computation of a "beta" for a bond (or a bond portfolio) that would indicate the percent change in price for the bond (or a bond portfolio) for a one percent change in price for a bond market series, one would expect a very high correlation between the beta for the bond (or the bond portfolio) and the bond's duration or the bond portfolio's duration. The point is, duration is a very good proxy for the interest rate risk for the bond or the bond portfolio.

Because of this relationship between duration and interest rate risk some investigators, including Wagner and Tito [36], have suggested that investigators should consider the construction of a bond market line using duration as the measure of risk. Specifically, the vertical axis has the realized rate of return on bond portfolios, while the horizontal axis would specify the average duration of the portfolios being examined. The market portfolio used would be some aggregate market

TABLE 7

SAMPLE OF GOVERNMENT BONDS USED TO CONSTRUCT  
MATURITY YIELD CURVE AND DURATION YIELD CURVE  
(as of November, 1978)

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<u>Bond Description</u>		<u>Yield to</u>		
<u>Coupon</u>	<u>Maturity</u>	<u>Maturity</u>	<u>Maturity</u>	<u>Duration</u>
7 7/8%	5/79	9.41%	.5	.500
7 1/8	11/79	9.68	1	.982
8	5/80	9.30	1.5	1.443
7 1/8	11/80	9.18	2	1.897
7 1/2	5/81	9.03	2.5	2.322
7 3/4	11/81	8.89	3	2.729
9 1/4	5/82	8.76	3.5	3.072
7 7/8	11/82	8.76	4	3.499
7 7/8	5/83	8.62	4.5	3.865
7	11/83	8.71	5	4.274
7 7/8	5/86	8.63	7.5	5.762
7 5/8	11/87	8.64	9	6.585
8 1/4	5/88	8.67	10	6.724
8 3/4	11/88	8.71	11.5	7.586
8 5/8	11/93	8.71	15	8.664
7	5/98	8.41	19.5	10.278
8 1/2	5/99	8.66	20.5	9.963
8 1/4	5/05	8.62	26.5	10.771
7 7/8	11/07	8.80	29.0	11.061
8 3/4	11/08	8.97	30.0	10.845

Figure 2  
MINORITY YIELD CURVE - GOVERNMENT BONDS  
(as of November, 1978)

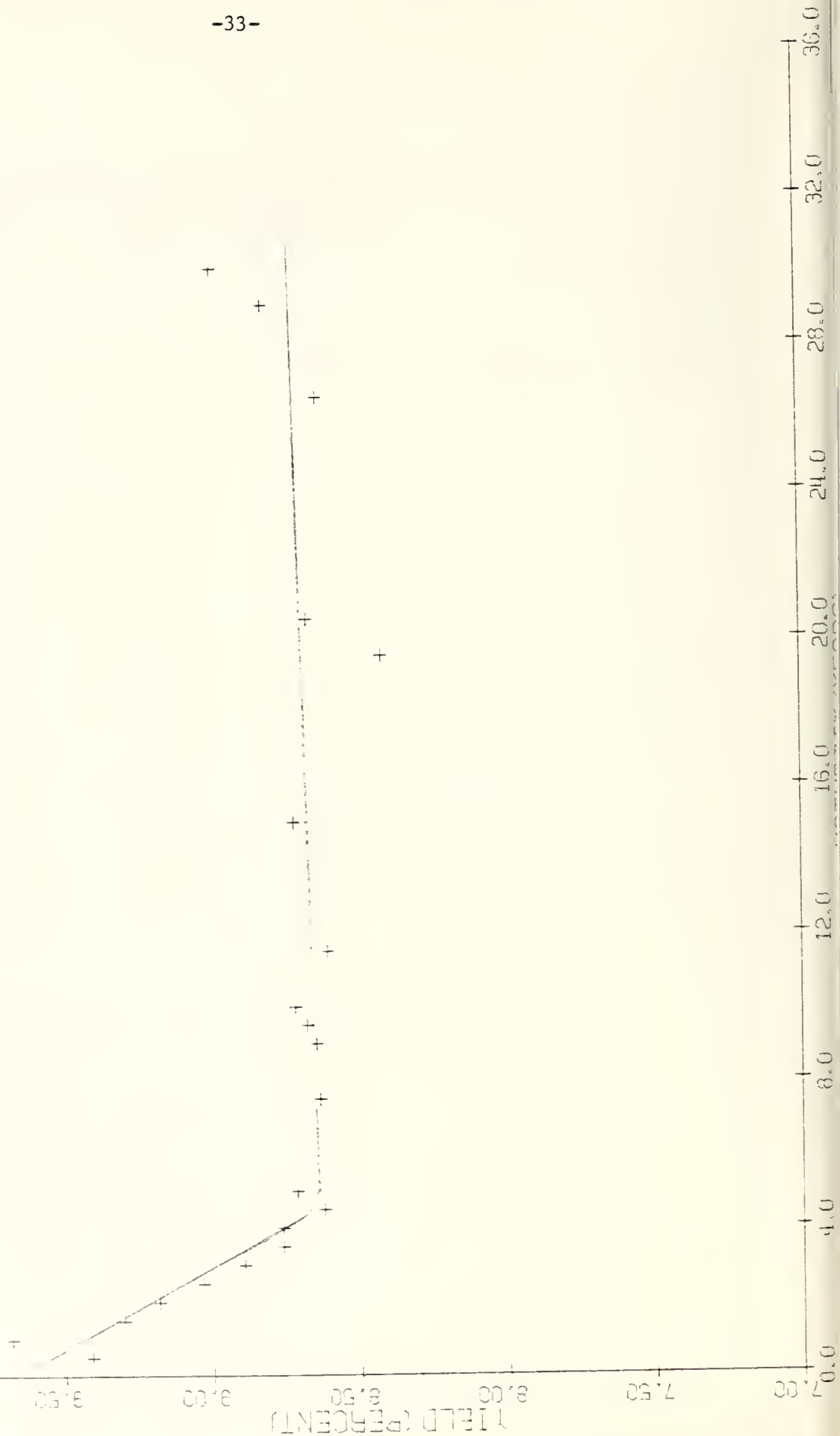
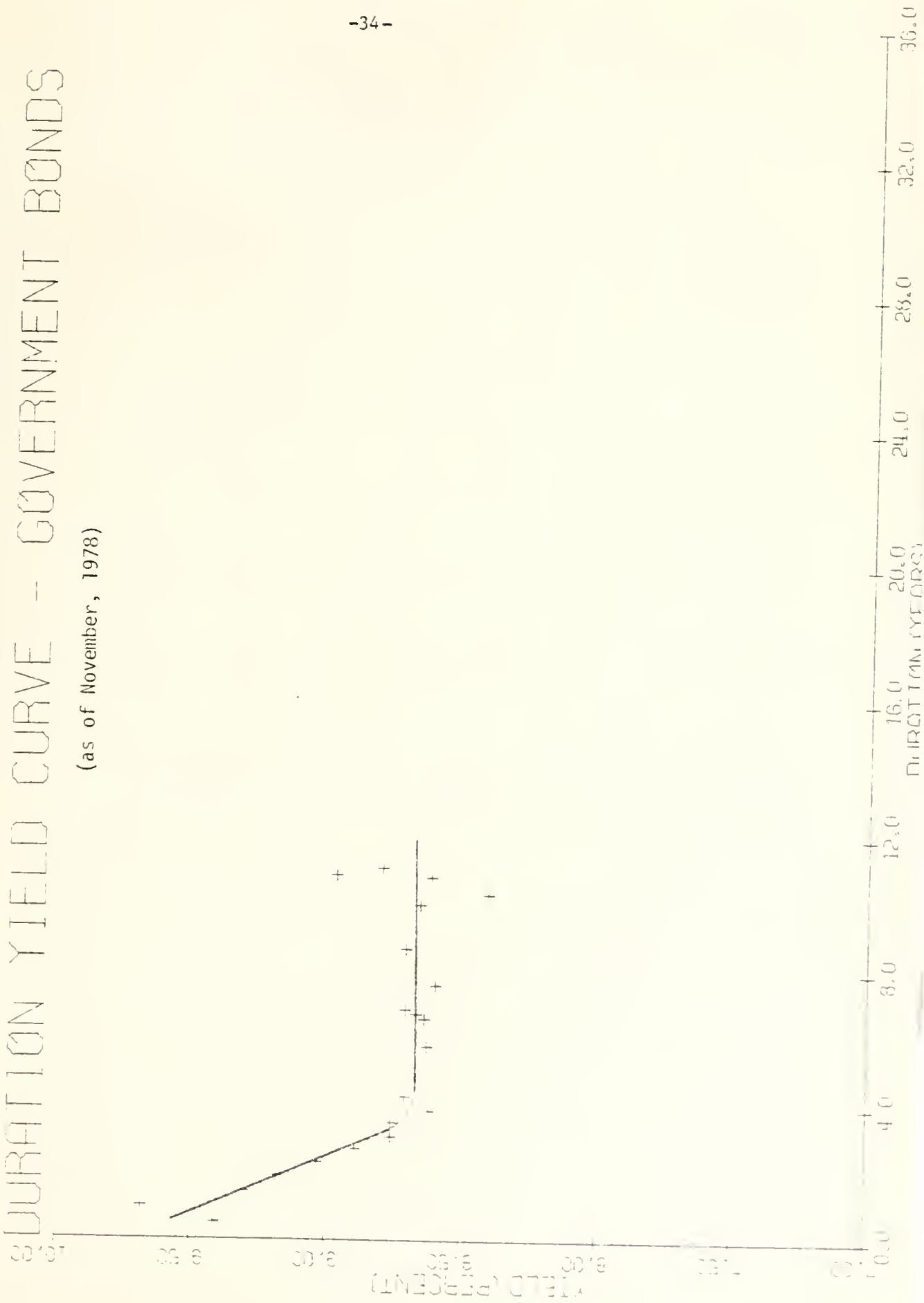




Figure 3

# DURATION YIELD CURVE - GOVERNMENT BONDS

(as of November, 1978)



series like the Solomon Bros. High Grade Bond Series or the Kuhn Loeb Bond Index. The graph would appear as in Figure 4.

While the concept of a bond market line is very appealing, the specification suggested has one major drawback--it does not allow for differences in the risk of default. Because duration indicates bond price volatility caused by changes in market interest rates duration is a good proxy for interest rate risk. Unfortunately, the bond market line that is constructed to take account of interest rate risk does not consider differences in default risk. Because one would expect a difference in the level of yield because of differences in default risk one would expect a series of bond market lines--a different line for every default class (i.e., one for government bonds, another for AAA rated bonds, a third line for AA rated bonds, etc.). An ideal example of such a multiple set of bond market lines would be as shown in Figure 5 (although the alternative market lines would not necessarily have to be completely parallel as shown). Theoretically, the difference between the bond market lines should reflect the default risk premium.

In addition to the hypothetical ideal bond market lines, we have derived a set of actual bond yield curves using rated public utility bonds. Note that the AAA rated duration yield curve in Figure 6 is downward sloping similar to the government bond curve. In contrast, the AA rated and A rated yield curves in Figure 7 and 8 have small positive slopes. Although space does not permit a discussion of the reason for the differing slopes (see Van Horne [36] or Malkiel [32]), the important point is that these differences along with the differences in the general level of yields for the alternative rated bonds means

Figure 4

Example of a Bond Market Line

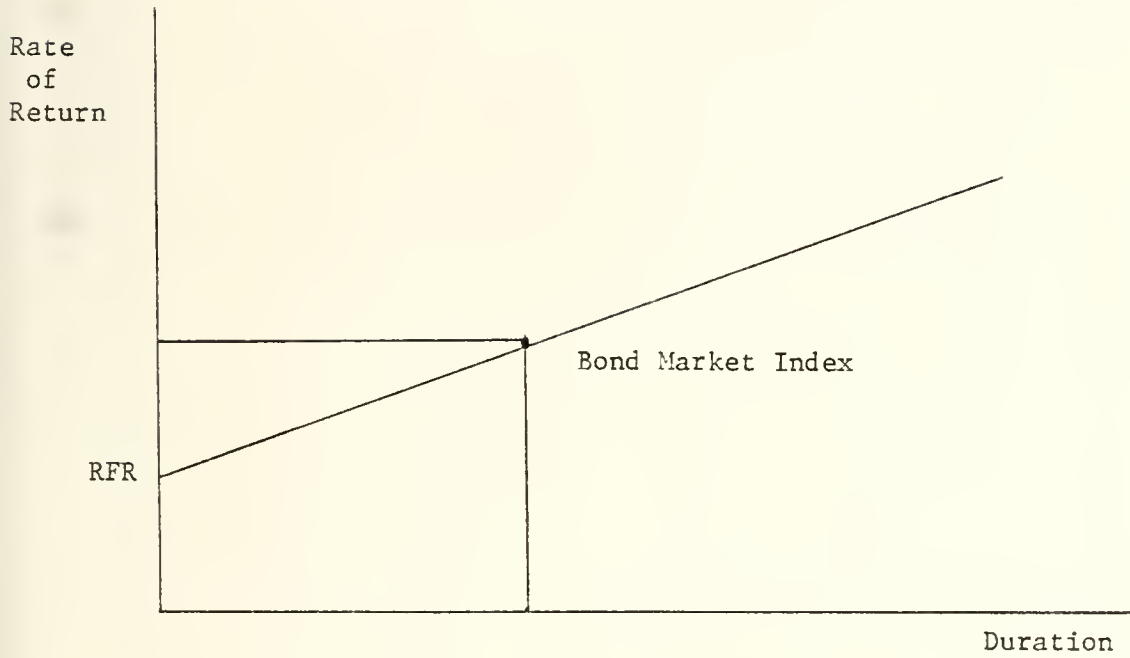


Figure 5

Example of Multiple Bond Market Lines  
For Bonds with Different Default Risk

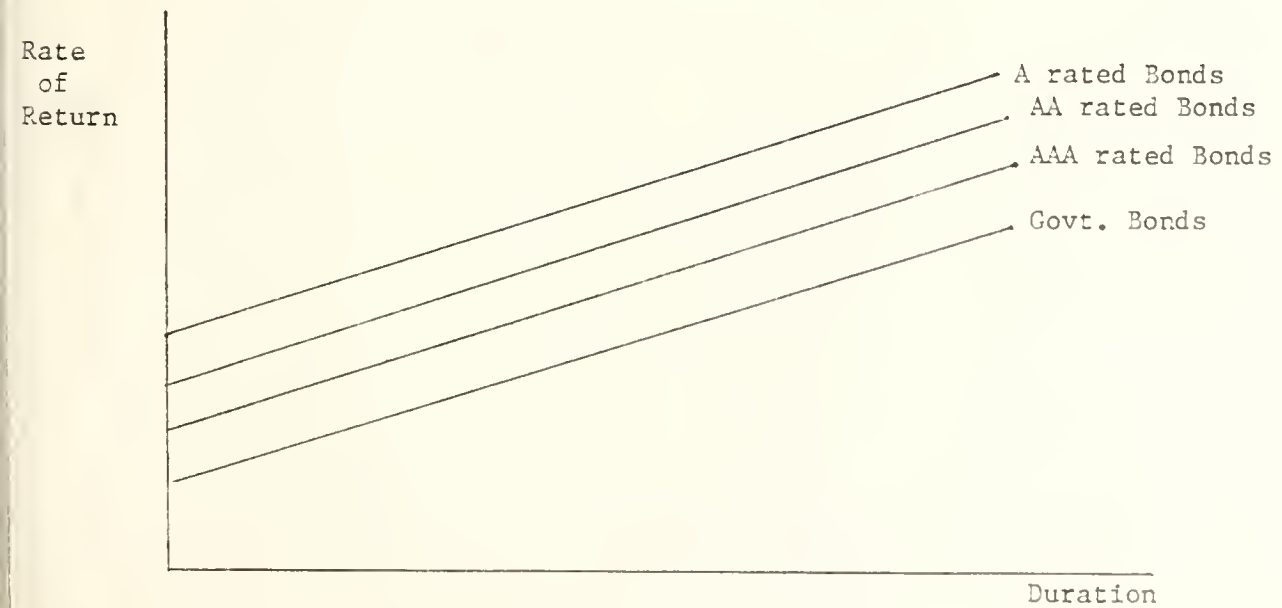


Figure 6

# DURATION YIELD CURVE - AAA PUBLIC UTILITY BONDS (as of November, 1978)

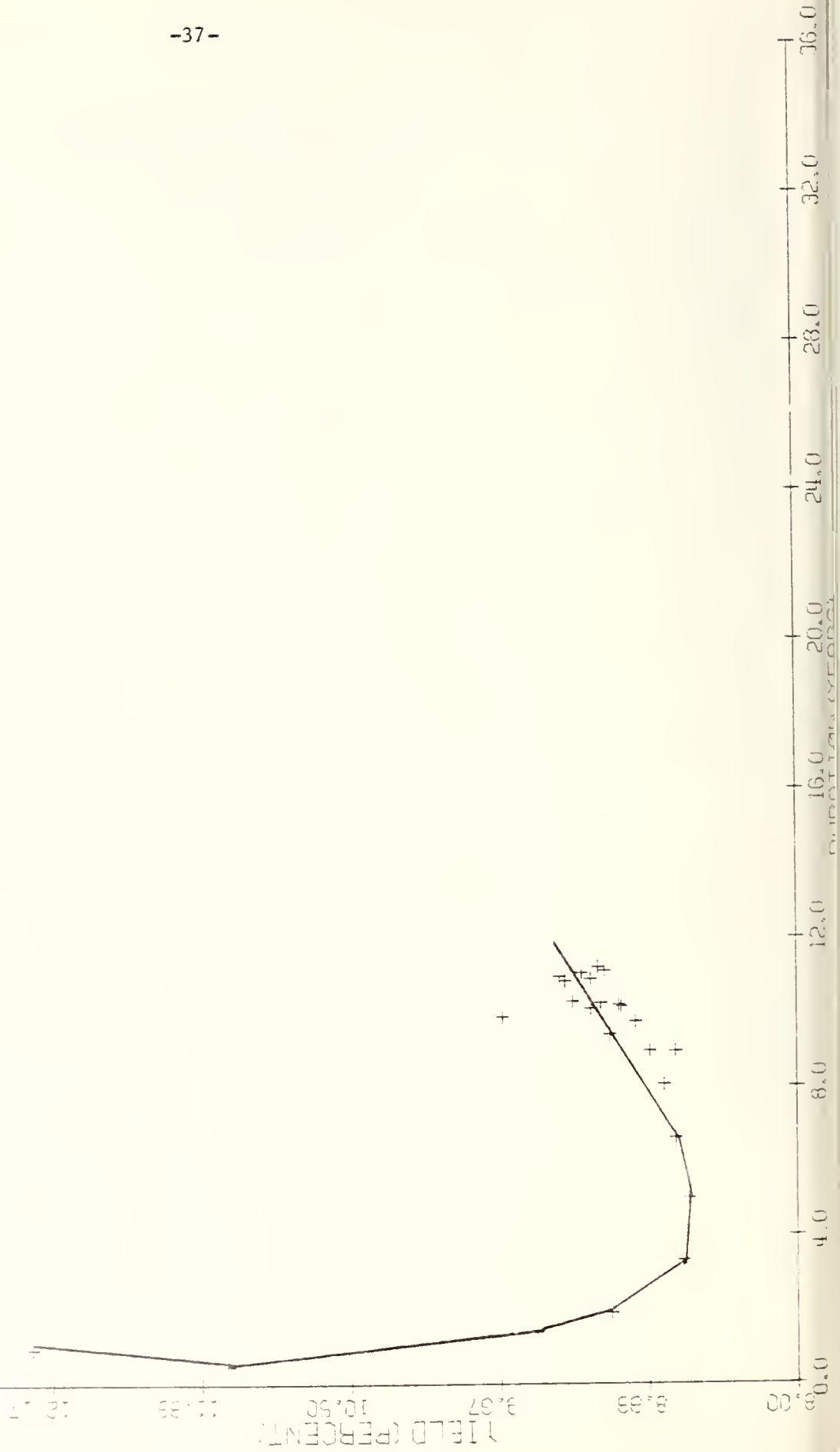


Figure 7

# DURATION YIELD CURVE - HA PUBLIC UTILITY BONDS

(as of November, 1978)

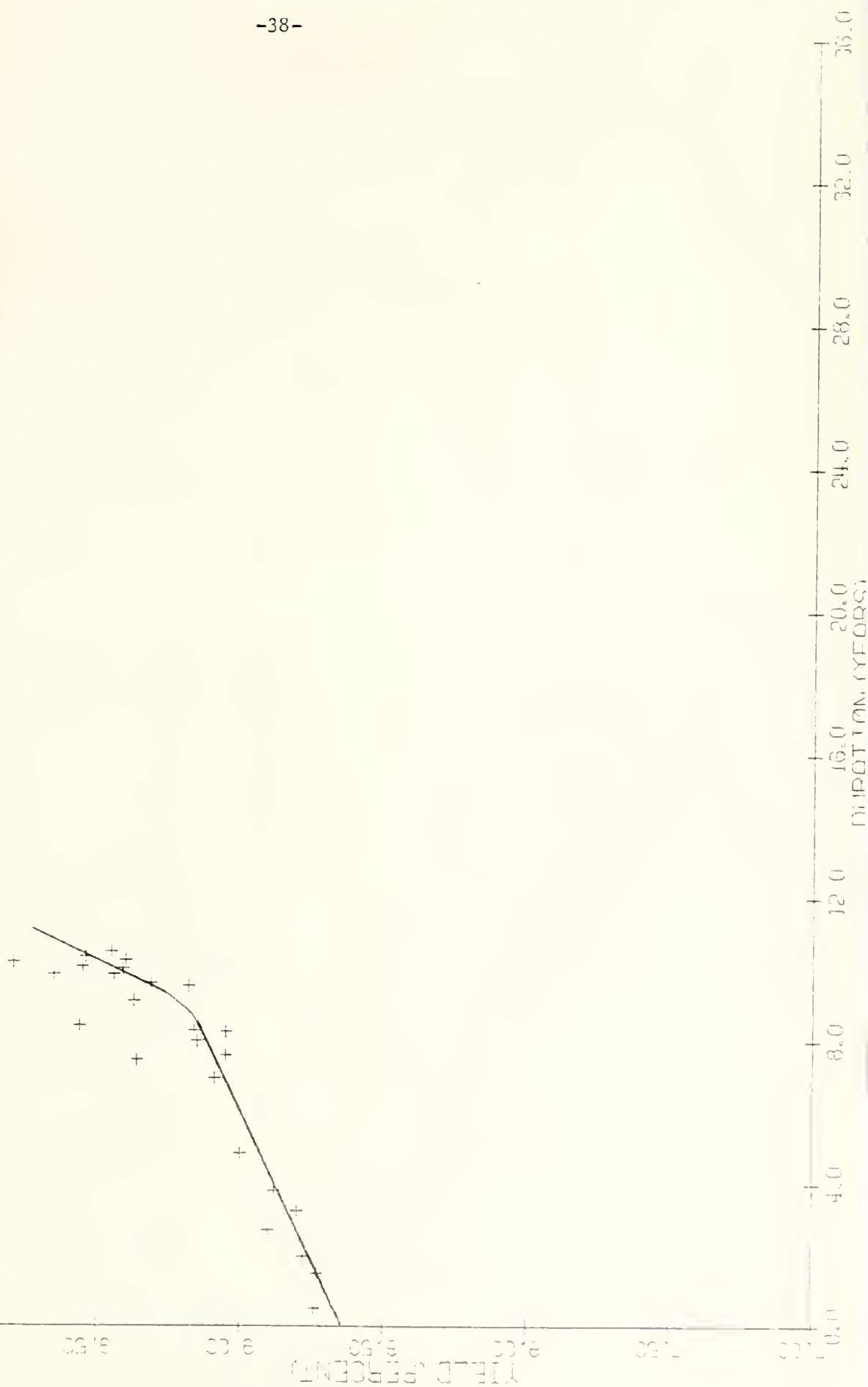
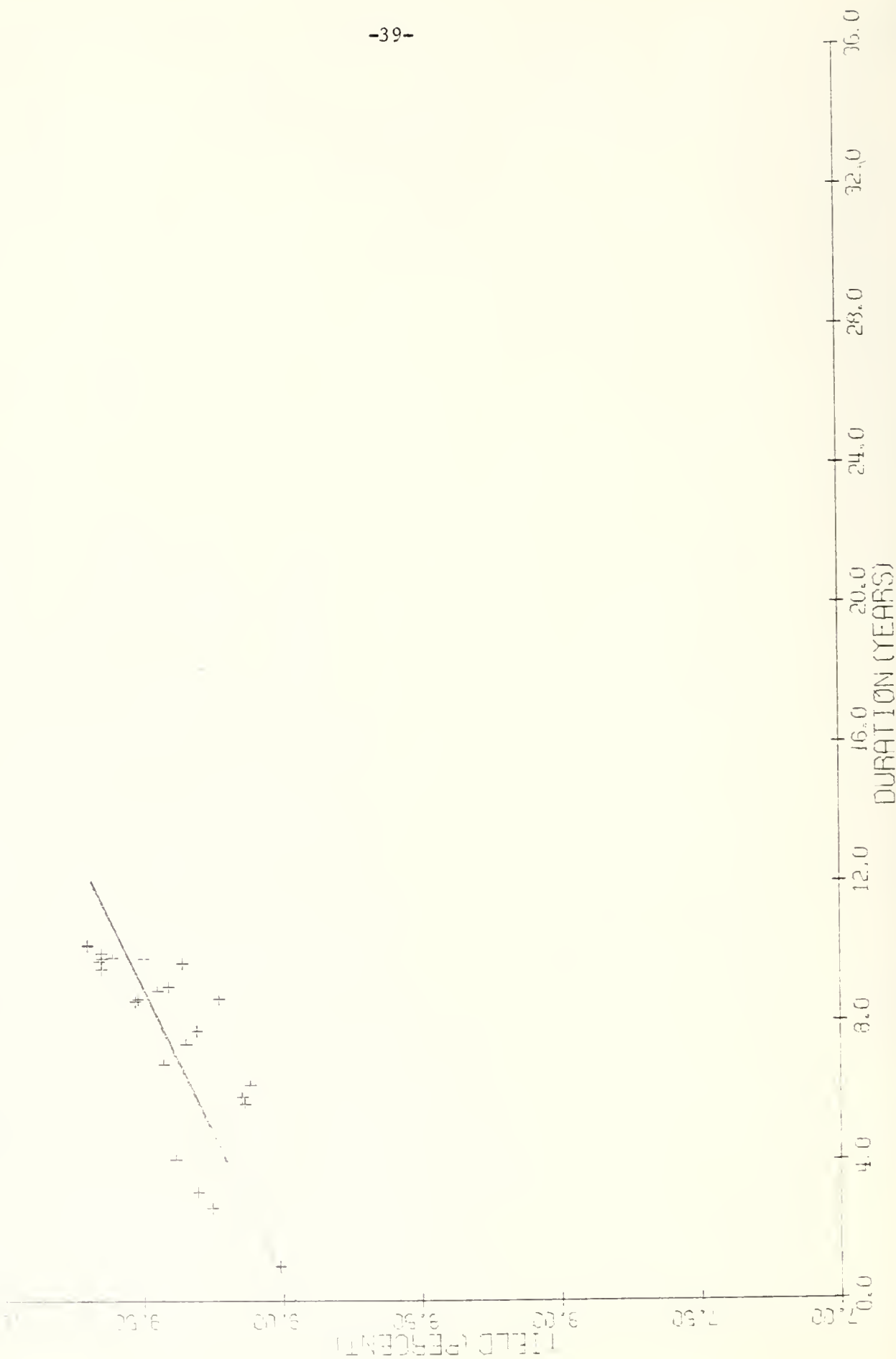


Figure 8  
DURATION YIELD CURVE - A PUBLIC UTILITY BONDS



that attempts to evaluate bond portfolio performance for portfolios with different average ratings using one bond market line that only considers interest rate risk is very questionable.

#### DURATION AND COMMON STOCKS

Although the bulk of the literature on duration has applied the concept to bonds, it is applicable to any investment flow including common stocks. It is important to recognize this because once it is acknowledged that duration can be computed for alternative common stocks, the other properties of duration are likewise applicable and can be considered in the valuation of common stocks and in stock portfolio management.

#### Computation of Common Stock Duration

The difficulties in computing the duration for a given common stock arise because of the several unknowns involved in the cash flows and the discount rate. In the case of high-grade bonds, the analyst knows the timing and amount of the interim cash flows based upon the coupon rate and the final cash flow from the principal at maturity. Also, the discount rate (using the Macauley definition) is the prevailing yield to maturity for the bond. In contrast, in the case of common stock, the interim cash flows would be the expected future dividend payments which are uncertain in amount. Further, the timing of the final cash flow is theoretically at some very distant unknown point since common stock is considered to have perpetual life. Further the amount of the final cash flow is also unknown. Finally, the discount rate used should be the prevailing required rate of return on the

security which in the case of common stock is likewise an estimate based on other estimates in the stock valuation model. In the standard dividend valuation model the required return is the  $K_i$  as follows:

$$P = \frac{D_1}{K_i - g_i}$$

so 
$$K_i = \frac{D_1}{P} + g_i$$

In summary, it is much more difficult to compute the duration for common stock because the amount and the timing of the cash flows are unknown and the appropriate discount rate is uncertain. Still, assuming that the analyst is willing to make the necessary estimates, it is possible to compute the duration for alternative common stocks. Clearly, the duration for alternative stocks can differ substantially depending on the estimates of cash flows and their timing. Notably, these differences in computed duration should affect the valuation of these stocks and the management of common stock portfolios.

To gain an appreciation of the problems and effects of different characteristics of stocks on the stock's duration, consider the following examples that progress from short-term, stable payment investments to long-run growth companies. We will also consider the effect of different estimates of  $K_i$ . For computational simplicity, it is assumed that all dividends are paid once a year at the end of the year.

Example 1. A common stock currently selling for \$20 pays \$1/year dividend and is expected to be sold at the end of five years for \$25. Alternative  $K_i$ 's are .08, .12, .16.

Example 2. The current price is \$20, the stock is expected to pay \$1/year for five years, \$1.20 for the subsequent five years, and is



expected to be sold at the end of 10 years for \$30.  $K_i$  equals .08, .12, .16.

Example 3. The current price is \$20 a share, the expected dividend is \$1.00/year for 20 years and it is anticipated that the stock will be sold for \$25 at the end of 20 years.  $K_i$  equals .08, .12, .16.

Example 4. The current price is \$20 a share, the expected dividend stream is \$0.50/year for three years, \$0.70/year for three years, \$0.90/year for four years, \$1.20/year for four years, \$1.50/year for four years, \$1.75/year for two years, and it will be sold after 20 years for \$40.  $K_i$  will be .08, .12, .16.

The computed durations for these alternatives are contained in Table 8. Note that all these durations are specific to the estimates made regarding the amount and timing of cash flows and the required rates of return and the duration could vary substantially between investors because of differing estimates. The purpose of these examples was to demonstrate the impact of differing dividend streams and selling prices.

The first stock indicates the effect of a short time horizon, a reasonable dividend and a small price increase. The second example extends the horizon and assumes some growth in the dividend stream and the price. The third and fourth examples both assume a 20 year holding period but differ in terms of the growth in the dividend stream and the ending price. Number three has a stable dividend throughout and little price change, while number four has a great deal of growth in the dividend stream and the price. A comparison of examples three and four indicate that with a smaller beginning dividend and subsequent growth,

TABLE 8

DURATION FOR ALTERNATIVE COMMON  
STOCK EXAMPLES

	$K_i$		
	.08	.12	.16
Example 1 Duration	4.591	4.549	4.505
Example 2 Duration	8.318	7.996	7.641
Example 3 Duration	12.263	10.364	8.630
Example 4 Duration	15.023	13.432	11.717

the duration increases substantially. Specifically, although both examples assume a holding period of 20 years (term to maturity), the duration of the growth stock is 23 percent longer than the stable income stock at 8 percent and 36 percent longer at 16 percent. The obvious implication is that growth stocks have longer durations than stable high dividend paying securities. Consistent with the bond discussion, the longest duration stock would be a high growth zero dividend stock that did not pay any current dividend, but was acquired on the expectation of large future capital gains. In such an instance the duration for the stock would expand the investment horizon. Similar to the bond discussion, an increase in the discount rate causes a decline in the computed duration.

A very important implication is that because growth stocks have longer durations than other common stocks growth stocks will be more volatile than other common stocks. In terms of modern portfolio theory, growth stocks on average should have higher betas than other common stocks. One of the first authors to consider the duration of common stock was Durand [12] who emphasized the long duration possibilities of growth stocks. Subsequently Malkiel [33] likewise discussed the long duration of growth stocks and specifically noted the effect this longer duration would have on their relative price volatility. Haugen and Wichern [21] discuss the interest rate sensitivity of numerous financial assets including common stocks. Probably the most complete direct analysis in this regard was by Boquist, Racette and Schlarbaum [7] who derived the specific relationship between the duration of a security and its beta and also the formula to compute the duration for common

stock using the basic dividend valuation model which is:  $V = d_1 / (K_i - g_i)$  where  $V$  is the total value of the common stock;  $d_1$  is the next period's dividend;  $K_i$  is the required rate of return on the stock; and  $g_i$  is the expected growth rate of dividends for the  $i$ th unit. It is shown that duration ( $D_i$ ) is equal to

$$D_i = \frac{1 + K_i}{K_i - g_i} \quad (\text{for discrete compounding})$$
$$= \frac{1}{K_i - g_i} \quad (\text{for continuous compounding})$$

Using the continuous compounding formula, the effect of differences in  $K$  and  $g$  can be shown. Consider the influence of the combinations of  $K$  and  $g$  on duration shown in Table 9.

Obviously, duration is determined by the spread between  $K$  and  $g$ -- i.e., the larger the spread, the lower the duration. Therefore, with an increase in the growth rate and all else the same, there will be an increase in the duration for a stock. In contrast, if one assumes an increase in  $K$  (e.g., due to inflation) without a commensurate increase in the firm's growth rate, there will be a decrease in duration. A note by Livingston [30] extended the Boquist, et. al. results by introducing the duration of the market portfolio. The extension indicated that the risk for a stock depended not only on the rate of growth (i.e., high growth rate, high risk), but also on the covariance between changes in the firm's growth and the market's growth (high covariance of growth, high risk).

TABLE 9

ESTIMATED DURATION FOR COMMON STOCKS  
UNDER ALTERNATIVE K AND g ASSUMPTIONS

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K	g	D*	K	g	D*
.10	.04	16.7	.14	.06	12.5
.10	.06	25.0	.14	.08	16.7
.10	.08	50.5	.14	.10	25.0
.12	.04	12.5	.16	.06	10.0
.12	.06	16.7	.16	.08	12.5
.12	.08	25.0	.16	.10	16.7
.12	.10	50.0	.16	.12	25.0

\*All computations use the continuous compounding formula =  $D_i = \frac{1}{K_i - g_i}$

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## SUMMARY AND CONCLUSION

### Summary

The concept of duration was rediscovered about ten years ago and has received substantial attention in the recent academic literature because of its usefulness in bond analysis and bond portfolio management. The purpose of this paper has been to describe this measure, show how it is computed, and demonstrate the effect of coupon, maturity, and the market yield. Subsequently, the relationship between duration and bond price volatility was discussed and the implication of this for active bond portfolio management was considered. A significant recent contribution is the recognition that it is possible to immunize a bond portfolio from interest rate risk under certain conditions by matching the investment horizon for the portfolio and the portfolio duration. There is also a consideration of how duration can be used in constructing yield curves and a set of bond market lines for bonds of differing default risk. Finally, we considered the potential estimation problems involved in computing duration for common stocks, the wide range of potential estimates and the implications of these differences in duration on the risk of the stocks especially as it relates to growth stocks.

### Conclusion

Duration has been rediscovered and has received wide acclaim because it has numerous useful applications for bond analysis, bond portfolio management, and equity analysis. Therefore, it behooves bond analysts, bond portfolio managers, and equity analysts to become familiar with the measure and its many uses. Hopefully this paper has helped in this regard.

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